Before starting this essay, I had no idea what I was going to write about. I first thought about statistics in a gambling show called *Kakegurui*, but I kept struggling with the math. I also considered writing about the shows *Cells at Work* or *Dr. Stone*, but those are more science-oriented. Then, I saw my tenor trombone, rarely practiced with, propped in my peripheral vision. My mind proceeded to race with questions about air, resistance, travel distance, and how these factors combined to produce music. After thinking about these questions, I decided to start measuring, testing, and playing to learn more about my trombone.

First, let's cover some brief history of the trombone. Before trombones, instruments called "sackbutts" were used in the 15th century. They were somewhat smaller, but similar in shape and use. As trombones developed, they changed in shape and size, and some even acquired extra tubes/pedals to play with.

Without knowing the history, I first began learning tenor trombone 8 months ago. I learned that, basically, the speed of the air determines the note's pitch, or how high/low it is. Additionally, the quantity of air determines the note's dynamics, or how loud it is. Air is manipulated by the player's lips and lungs.

Human lungs can hold a maximum of around 6 liters of air. I tried to fill my lungs with as much air as possible, and then played the same note on my trombone at the same dynamic. I timed myself playing a note in 1st position, and then found the average time I could hold the note after repeated attempts.

With approximately 6 liters of air I could hold the note:

Bb2 for an average of 17 seconds.

6 L/17 seconds = ~0.353 L of air per second = ~353 mL of air per second

F3 for an average of 31 seconds.

6 L/31 seconds = ~0.194 L of air per second = ~194 mL of air per second

Bb3 for an average of 28 seconds

6 L/28 seconds = ~0.214 L of air per second = ~214 mL of air per second

D4 for an average of 17 seconds

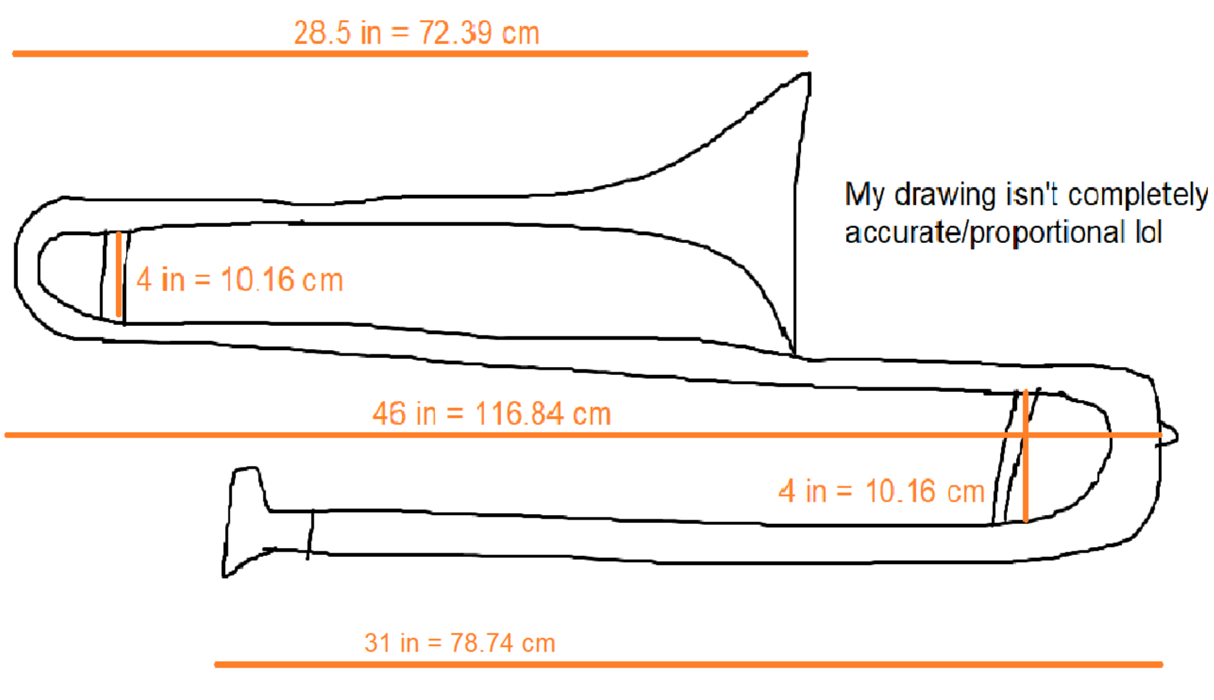
6 L/17 seconds = ~0.353 L of air per second = ~353 mL of air per second

F4 for an average of 11 seconds

6 L/11 seconds = ~0.545 L of air per second = ~545 mL of air per second

Because different notes require different air speeds, I tried to determine the trombone's distance from the mouthpiece to the bell. The distance combined with the time would give me the air speed for each note.

With a tape measure, I measured my trombone in first position, and got these numbers.



The curved ends are half of a circle, so I will calculate the distance of the curved parts by finding half of the circumference.

Circumference = π * 2 * radius = π * diameter

Circumference = 10.16 cm * π = ~31.919 cm

There are two semicircles with the same diameter, so these can combine to make a full circle.

To find the total distance, I added all of the measurements.

78.74 + 116.84 + 72.39 + 31.919 = 299.889 cm

The length of the internal tubing in the trombone (in first position without the tuning slide adjusted at all) is 299.889 cm.

According to the article [Experiment] Let's make a straight trombone from Yamaha, tenor trombone tubing is around 270 cm, so my measurements may have been different/incorrect.

Now, I can find the speed of my air in the trombone by using the trombone's measurements and the seconds I could hold the note.

Speed = distance ÷ time

For Bb2:

Speed = 299.889 cm ÷ 17 seconds = ~17.641 cm/s

For F3:

Speed = 299.889 cm ÷ 31 seconds = ~9.674 cm/s

For Bb3:

Speed = 299.889 cm ÷ 28 seconds = ~10.710 cm/s

For D4:

Speed = 299.889 cm ÷ 17 seconds = ~17.641 cm/s

For F4:

Speed = 299.889 cm ÷ 11 seconds = ~27.263 cm/s

I realized that something was wrong. Slower air is supposed to be for lower notes, and faster air is for higher notes. However, Bb2 has faster air compared to F3, but is a lower note. After thinking about the problem, I realized that Bb2 uses more air per second. This means that it uses more air in a shorter amount of time. I tried to factor in the amount of air into the equation as well. The calculation was for the speed of a certain amount of air rather than the speed of the air itself.

For Bb2:

Speed of 353 mL/s = 299.889 cm ÷ 17 seconds = ~17.641 cm/s

17 seconds * 353 mL is very close to 6 liters.

For F3:

Speed of 194 mL/s = 299.889 cm ÷ 31 seconds = ~9.674 cm/s

31 seconds * 194 mL is very close to 6 liters.

For Bb3:

Speed of 214 mL/s = 299.889 cm ÷ 28 seconds = ~10.710 cm/s

28 seconds * 214 mL is very close to 6 liters.

For D4:

Speed of 353 mL/s = 299.889 cm ÷ 17 seconds = ~17.641 cm/s

17 seconds * 353 mL is very close to 6 liters.

For F4:

Speed of 545 mL/s = 299.889 cm ÷ 11 seconds = ~27.263 cm/s

11 seconds * 545 mL is very close to 6 liters.

This makes more sense. Higher notes use less air that moves faster, while lower notes use more air that moves slower. This explains why the note Bb2 appeared to be faster than F3, which is higher. It is not that the air is moving faster, it is just that more air is moving, which makes the number higher. After F3, the numbers appeared to act normally, where higher notes would have higher air speeds.

This can be explained by the player's mouth and lungs. As a note gets higher, the player makes their mouth smaller to speed up the air. The amount of force is pushed through different mouth sizes to change the air's pressure, which in turn changes the note's pitch.

Pressure = force ÷ area

When I make my mouth smaller, air is pushed into a smaller space, making the pressure greater. The air travels faster, and the note sounds higher. When I make my mouth larger, the opposite happens, and the note sounds lower. I was thinking of measuring my mouth for different notes, but I could not accurately measure myself.

To explain why Bb2 appeared to have a faster speed, my embouchure was larger. Although this made the air slower, it also made more air leave my lungs quickly, which did not let me hold the note for too long. This also explains why I tend to struggle with holding low notes.

Another factor affecting trombone sound is bore, which is the inner diameter of the inner slide.

My trombone bore = 0.500" = 1.27 cm

Now that I have my trombones bore, I can calculate the approximate volume of the tubing inside my trombone with the diameter and the total length of the trombone.

Internal tubing length = 240.199 cm

Bore = 1.27 cm

The volume of the internal tubing is basically like a long cylinder.

Cylinder Volume = π * radius^2 * height = π * (diameter/2)^2 * height

Volume of internal trombone tubing = π * (1.27/2)2 * 240.199 = ~304.277 cubic cm

In first position without the tuning slide adjusted, air must travel 240.199 cm, through a pipe with a volume of 304.277 cubic cm. However, when the tuning slide and slide are adjusted, the length of the trombone changes as well, and distance/volume also change.

I measured the lengths of the trombone again, but with each slide position (1 through 7) to see how far air must travel. I adjusted the tuning slide to be 1.27 cm (0.5 in) out

First position = 78.74 + 118.11 + 73.025 + 31.919 = 301.794 cm

Second position = 86.36 + 123.19 + 73.025 + 31.919 = 314.494 cm

Third position = 99.06 + 134.62 + 73.025 + 31.919 = 338.624 cm

Fourth position = 106.68 + 142.24 + 73.025 + 31.919 = 353.864 cm

Fifth position = 114.3 + 151.13 + 73.025 + 31.919 = 370.374 cm

Sixth position = 124.46 + 161.28 + 73.025 + 31.919 = 390.684 cm

Seventh position = 142.24 + 180.34 + 73.025 + 31.919 = 427.524 cm

As the slide extends further, the note’s pitch gets lower. I was not sure as to why, but after some thinking, I had an idea. Pitch is how high or low a note is, and it is the same as frequency, which describes the amount of sound waves in a certain amount of time. As the trombone tubing extends, perhaps the air loses energy as it travels, which causes its frequency to drop. This could possibly explain the lower pitches that come from extended slide positions.

After studying, measuring, and just playing with my trombone, I got a cool insight into some of the math involved with trombones. Music, in general, is a field that is pretty far from math. It was pretty fascinating to see how they connected, despite their differences.

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