Myth Busting: Teleportation Edition

Teleportation isn't a thing, the only place where it's possible is in the world of movies and TV ……..right? Although teleporting humans is sadly impossible (currently!) there is such thing as quantum teleportation which allows quantum information to be teleported.

But first, a crash course in the basics of quantum computing. Quantum computers are unlike classical computers – your everyday computer – as they harness a type of physics and its properties called quantum mechanics. This means that they are capable of completing tasks that a classical computer can either not do or doing said task exponentially faster which is why there is so much research into them. Within a classical computer there can only be two states, on or off, which are 1 and 0 respectively. These states are called bits and their value is a method of storing information, vital for a computer to function. However, in a quantum computer these states are called qubits and the notation for them is $|0>$ ($\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in vector state) and $|1>$ ($\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ in vector state). Unlike a classical bit, qubits can be in both states at the same time and only when measured will they collapse into a state of either $|0>$ or $|1>$. This is called superposition and is one of the special properties a quantum computer is able to harness due its use of quantum mechanics. We write a quantum super position like this:

$$|\varphi> = \alpha |0> + \beta |1>$$

Where φ is a quantum state and α^2 is the probability of measuring $|0>$ and β^2 is the probability of measuring $|1>$. Once measured the quantum state (φ) either equals $|0>$ or $|1>$.

Gates can be applied to qubits in order to change the state of them and the application of quantum gates means a quantum circuit is created. There are four main types of gates used in teleportation all of which can be displayed as a matrix.

Gate	Matrix	What does it do?
Pauli – X (X)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	The X gate is a bit flip gate which means that any qubits in the $|0>$ state go to the $|1>$ state and vice versa.
Pauli – Z (Z)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	The Z gate is a phase gate and changes the sign if there is a $|1>$ qubit. The $|0>$ qubit stays the same but if the Z gate is applied to the $|1>$ then it goes to $-|1>$ and likewise $-|1>$ goes to $|1>$
Hadamard	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	The Hadamard gate creates a fifty – fifty superposition so the quantum state will look like this: $$|\varphi> = \frac{1}{\sqrt{2}}\left|0> + \frac{1}{\sqrt{2}}\right|1>$$

CNOT	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	When the CNOT gate is applied, if the control qubit (q_0) is |0> then nothing happens but if it is |1> then the target qubit (q_1) is flipped so |0> would go to |1> and |1> would go to |0>

Another attribute of quantum mechanics used in quantum computing is entanglement. This is a phenomenon which occurs when two qubits are created in way so that when one is measured this determines the result of the other one no matter the distance between these two qubits – a very important feature needed for quantum teleportation. You create an entangled state by taking two qubits and applying a Hadamard gate and then a CNOT gate.

A visual representation of creating an entangled state
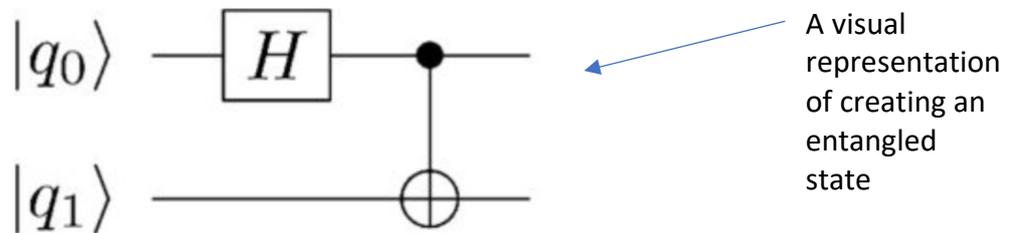
Bell states, used in quantum teleportation, are the 4 two-qubit states that are maximally entangled. This involves taking classical information, applying operations in order to produce the bell states:

Message	Bell State	Result
00	$\dfrac{\|00> + \|11>}{\sqrt{2}}$	$\|\beta_{00}>$
01	$\dfrac{\|10> + \|01>}{\sqrt{2}}$	$\|\beta_{01}>$
10	$\dfrac{\|00> - \|11>}{\sqrt{2}}$	$\|\beta_{10}>$
11	$\dfrac{\|10> - \|01>}{\sqrt{2}}$	$\|\beta_{11}>$

But how does all of this apply to teleporting? One of the requirements for quantum teleportation is that an entangled bell state must be split between the sender and receiver before teleportation takes place. This will involve the sender and receiver meeting up to create a bell state and then taking a qubit each from the state whilst they're entangled. These bell states are now used in the process of teleportation alongside a classical communication channel e.g. phone and two classical bits.

To start, the sender has the half of their bell state (in this example $\|\beta_{00}>$ will be used as the bell state) and another qubit of which they do not know the state of ($\|\varphi>$). Teleportation

happens after an input is passed through a quantum circuit and the input for this state would be:

$$|\phi> = |\varphi> |\beta_{00}>$$

Then the sender uses a CNOT gate where their part of the entangled bell state is used as the control and the quantum state $|\varphi>$ is the target. The state $|\varphi>$ also has a Hadamard gate applied to it which entangles the qubit needing to be teleported and the sender's portion of the bell state. The application of the Hadamard has an effect on the receiver's part of the bell state as they're entangled! This will effect what happens in the measurement stage.

The next step is for the sender to measure their qubits, causing them to collapse to their classical bit states. For this example and the bell state we've used, if the sender measures their state as 00 then the receiver is guaranteed to be in the state of the unknown qubit that was teleported due to entanglement.

Sender's Measurement	Receiver's Qubit State
|00>	$\alpha|0> + \beta|1>$
|01>	$\alpha|1> + \beta|0>$
|10>	$\alpha|0> - \beta|1>$
|11>	$\alpha|1> - \beta|0>$

However, if the measurement has produced any of the other results then some additional operations will need to be performed in order to achieve the state desired. To do this, the receiver needs to know how to correct it. This is done by sending the result of the sender's measurement, which is the value of their qubits, via a classical communication channel and depending on the measurement recorded the gates used will differ. The second measurement will have an X gate applied to it on order to flip the values of the qubits, the third has a Z gate performed on it so the sign switches from negative to positive and the last measurement in the table will have both an X gate and Z gate act on it. Now that the receiver has corrected their state, the quantum teleportation of the original, unknown qubit has taken place!

Quantum teleportation is an amazing resource however has some limitations and differs slightly to our initial thoughts when it comes to teleporting. It only allows for the teleportation of one qubit at a time and this does not happen instantaneously both of which contradict popular culture's portrayal of teleporting. It also requires meeting up to share a bell state and each time teleportation is performed a new bell state must be shared as when it is measured the state collapses.

Despite this, quantum teleportation allows for absolute privacy as the shared bell state is required and so its potential benefits for cybersecurity are immense. The mathematics and physics behind it is truly fascinating and brings together so many different concepts to create the beautiful method of quantum teleportation.

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Also I am taking part in the Qubit by Qubit course which teaches the basics of quantum computing, mechanics and algorithms which is where I learnt the majority of the information in this essay.