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The Mathematics behind Football and how it influences the Beautiful Game?

To some, Mathematics is the beauty of pure expression and to many, Football is regarded as the 'beautiful game'. It's fascinating how two arts which at first seem so polar, in reality share many characteristics, whether: it's the moments of brilliance demonstrated by genius' such as Andrew Wiles, when he proved Fermat's Last Theorem and Lionel Messi, as he dribbled past the whole of the Bayern Munich team in the Champions League to score, or the dedication and hard work displayed by professional athletes and high school teenagers like myself. Mathematics and Football have an interesting correlation. This connection is being recognized by the biggest football clubs in the world, who are using mathematical analysis to improve their performances, scouting and transfer efficiency. Therefore with mathematics growing more prevalent in football, I began to research how Mathematics influences football and whether it is for the better of the beautiful game?

Performance Analysis and Transfer Decisions

Recently Mathematics has been used to calculate the most cost-efficient way to win football games without spending excess money on players. A perfect example is Liverpool Football Club, who are owned by Fenway Sports Group. The company's foundations for success are based on buying players who can statistically perform just as well as a global superstar. Therefore after Liverpool won the UEFA Champions league, the Club World Cup and the Premier League(for the first time in 30 years!) in the space of two years, running a football club through mathematics and analytics gained a proven reputation.). However, to improve a football team you must analyze their current performances and this analysis is a key factor for the decision making of a football club owner like Fenway Sports.

The latest revelation in football analysis are expected goals; which are now used to analyse performances of players as well as teams as a whole. Expected goals also known as xG, is a measurement of the quality of each shot at goal in a game and the probability that a shot leads to a goal. Analytics companies such as Opta have analysed over 300,000 shots in order to calculate the likelihood of a shot being scored. The model takes into account the assist type (if the assist is a cross, a soft or a hard pass), if the ball is shot with your head or your foot, how big or crucial the chance is, and the angle and distance the shot is taken away from the goal. The expected goal value is a probability usually represented by a decimal. It helps coaches understand if a certain player or their team are converting their chances and if they are creating enough chances initially. Other ways in which teams are analyzed are through expected assists sequences and possessions (which conveys how far a sequence of pass advances the team's possession up the pitch).

All of these statistics and data available on each player, have an effect on which players each club buys and sells. The data is used to suggest how well a player would

function within the system at a club. But how reliable is the mathematics behind this data? For instance, a goalkeeper may seem the best in the league because he or she has made the most saves and has the highest save percentage per shot but this may be the case as the goalkeeper is playing for a lower tier club with a poor defense thus faces more shots with a greater number of opportunities to make saves. Furthermore, with all of these numbers and attempts of making football more efficient, is the mathematical analysis contributing to football becoming more 'dull and boring'. During the rise of football analysis, there has been a sudden decline in flair players (football players who entertain and mesmerize fans by producing moments of skill whilst dribbling past their opponents) on the world stage. Have the numbers made the game into a business of efficiency and functionality? Surely the reason why people love football is the unpredictability and imagination of the game? To this, one only needs mention Leicester City 2015/2016. A team, who had finished 14th in the Premier League in the previous season and had 5000/1 odds of winning the league in the 2015/2016 season, did the unimaginable. Although Mathematics doubted that it was highly unlikely for Leicester to win the league, in fact the very same Mathematics had also helped the Leicester City scouting team analyse the players, such as the likes of N'golo Kante ,who they would sign to help them win the league. Therefore, I believe that the growing prevalence of Mathematics within football is in reality attempting to close the gap between the smaller teams and the high spending teams as Mathematics has the ability of making football cost efficient.

Tactics

Football strategies are devised by managers and coaches in order to decipher whether their team should play with the liberty to attack or the rigid structure of defense. Usually football managers change or slightly alter their strategies depending on which team they are opposing. For instance the general assumption was that a team should play attacking football (try to score more goals) against a weaker opposition and should play defensively against a stronger opposition. Game theory can be used to find the optimum strategy in certain instances so your team has the best chance of scoring more points from a match. The optimum strategy is devised after a scout has calculated the chances of winning, drawing or losing if your team plays attacking or defensive football when the opposition attacks or defends. However, these percentage chances are hypothetical because scouts are unable to provide such accurate data. It is more important to see your team's strength relative to the oppositions strategy options. From this data you can calculate your team's expected points in the attack/defend game under the system where there are 3 points for a win, 1 point for a draw and 0 points for a loss. Below is an example of an expected points table where, W is the chance of winning and D is the chance of a draw. W_1, W_2, W_3 , W_4, D_1, D_2 , D_3 and D_4 would have different values as each chance of a win or a draw is subjective to each scenario. Losses are not taken into account as losses gain a team 0 points and 0 multiplied by x=0.

	Opposition attacks	Opposition defends
You attack	$3W_1+1D_1$	$3W_2+1D_2$
You defend	$3W_3+1D_3$	$3W_4+1D_4$

Another way in which mathematics demonstrates that it can help managers decide their game strategy is by an equation which can generally represent a football match, where any outcome is possible. Let's assume that if both teams attack the chance of a win is w, the chance of a draw is d and the probability of losing is l. Also when one team defends the probability of winning and losing is multiplied by p(the effectiveness of defense).Therefore, (1-p) represents the effectiveness of no defense. A weaker team should defend when:

$3pw+1d+w(1-p)+l(1-p)>3w+1d$

$w(1-p) + l(1-p)>3w-3pw$

$w(1+p) + l(1-p)>3w(1-p)$

$l>2w$

Therefore, from this equation we can deduce that a weaker team should defend only if their probability of losing when playing attacking football is twice that for defending. This equation conveys that attacking football usually yield the most points per game and this is reflected by the manager and teams styles in the modern day Premier League. Clubs owners have recently opted for more attack-minded managers rather than defensive managers. However, this equation will only prove to be useful if the manager is able to understand the relative strengths of his team and the opposition. Betting company' odds and the Premier League table can provide this information. Thus Mathematics is a useful tool to gain a strategical advantage on your opponent in football and consequently the game is becoming more heavily based on strategy than on the individual performance of players.

Geometry is embedded within football whether its parabola trajectory (rainbow like) of a football due to gravity or it's the pattern of passes from different angles. After they have decided their strategy, managers must create tactics for their player to fulfill their strategy. Most assume that football is mainly based on controlling the ball but football tactics are devised in order to both create and control space. For instance a manager must decide how his team should press (pressure) the opponent in order to regain possession by controlling space on the pitch. The key to pressing is narrowing down the opponent's (the player who has the ball) options and to try to enclose them from all angles towards your own goal. Though there are two main forms of pressing 'counter press', which is applied immediately after your team has lost possession, and 'deep press' which is applied when defending the final third. By accumulating and analyzing enough data, you can calculate the pattern for a successful press. For instance for a counter press it was calculated that the first player must press the opponent with the ball within 2.3 seconds and a second player must help press within 5.5 seconds. Pep Guardiola, one of the most successful football coaches of modern football, called this the six second rule and he timed his player to see whether they could regain possession within 6 seconds. For a successful deep press you must reduce the

speed at which the opponent, with the ball, is travelling towards the defense. This can be done by allowing one player to press the opponent whilst the rest of the defence blocks the passing options in the final third of the pitch.

To conclude there is a high correlation between mathematics and football. The mathematics behind football has and is continuing to revolutionize the game. Mathematics has improved football because it can be used by smaller clubs as an attempt to catch up and compete against the high spending elite football teams. There are many mathematical models which can help us understand and see the beauty of football from different perspectives. But the mathematics within football is only going to grow and advance even further. Football is already played on the field of mathematics but could a mathematician someday manage and lead a football team to glory?

Sources

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