



*THE RIEMANN*  
***HYPOTHESIS***

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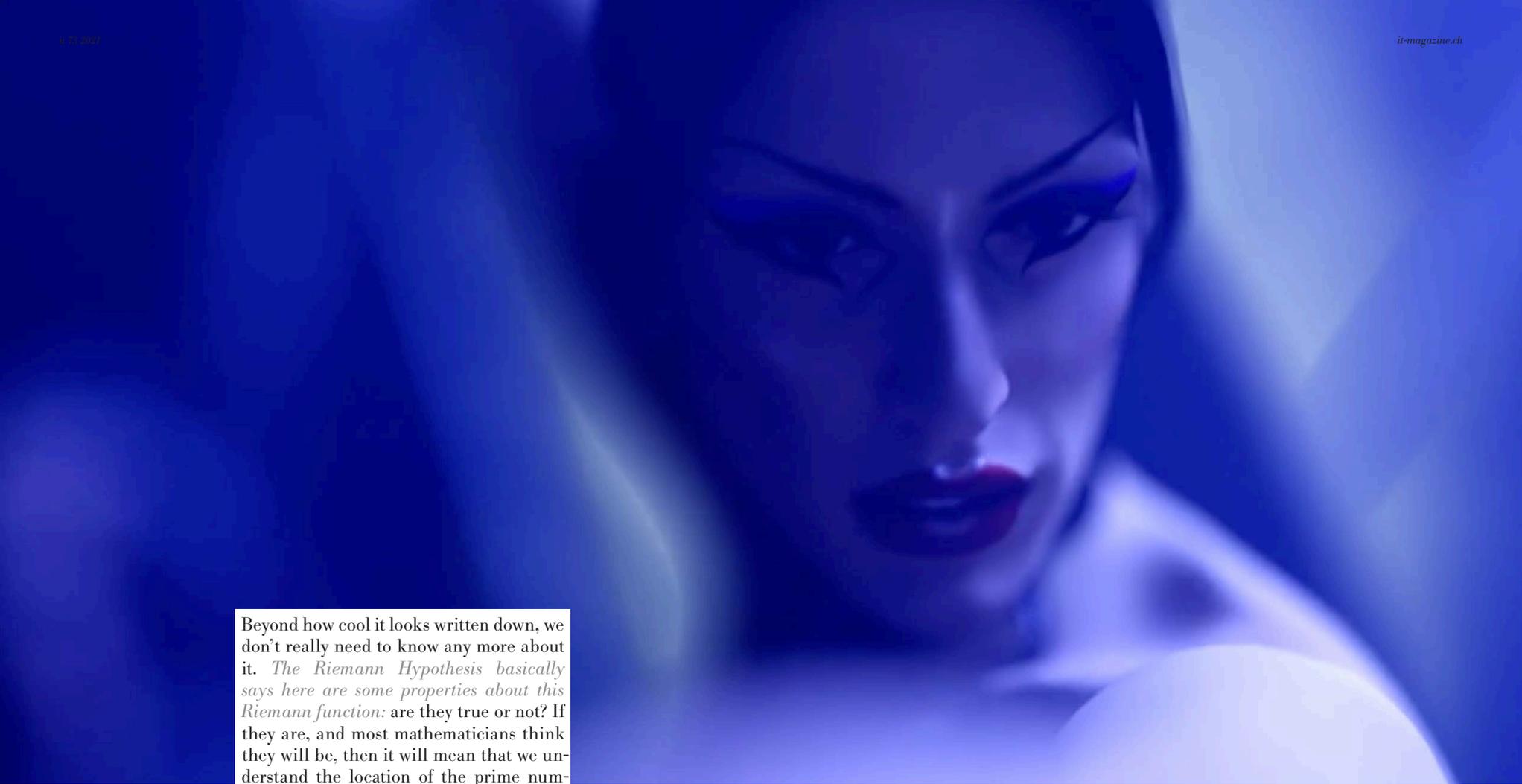
**T**he Riemann Hypothesis is a great place to begin our adventure exploring the untamed jungle of the Millennium Problems, as it's the only one of Hilbert's original 23 problems from 1900 that remains unsolved (*if you have no idea what I'm on about you had better go back and read my first article here – it's great trust me*).

The Riemann Hypothesis is both an oldie and a goodie. It was first formulated in 1859 by Bernhard Riemann and is still puzzling mathematicians over 150 years later. The hypothesis looks at prime numbers – *these are the ones that can only be made by multiplying themselves by one and have no other factors. Let's take the number 42, also known as the answer to the universe and everything in it. 42 has lots of factors,  $7 \times 6$ ,  $21 \times 2$ ,  $42 \times 1$ , okay it has six factors, but importantly it has more than two. Now what about the number next to it, 41. The only way to get this is by multiplying 41 times 1 – that's it. This makes 41 a prime number.*

Prime numbers are very special in maths because all numbers can be made from them. If I give you any number, you can break it down into smaller and smaller pieces with each one being a prime number. Back to our example of 42.  $7 \times 6$  is the same as  $7 \times (3 \times 2)$ .  $21 \times 2$  is the same as  $(7 \times 3) \times 2$ .  $42 \times 1$  is also  $7 \times 3 \times 2$ . You'll notice there's a pattern here, the building blocks to make 42 are 7, 3 and 2 which are all prime numbers. *This will be true for any number you can think of – give it a go: write down the factors, and then keep breaking down each factor into smaller and smaller factors and eventually you get down to prime numbers.* If DNA gives the building blocks of life, prime numbers give the building blocks of maths. While we now understand DNA pretty well – we can edit it and cause all kinds of cool things to happen, the same cannot be said for prime numbers. We know what they are and that all numbers can be written in terms of them, but that's pretty much the whole story. We really don't understand them very well, which brings us very nicely back to our Millennium Problem.

The Riemann Hypothesis talks about a very complicated function called the Riemann Zeta Function. I'll show you what it looks like because it's cool – and if you want to impress your friends, just say “*I know what the Riemann zeta function looks like*” and write the formula down. *Disclaimer:* the success of this tactic may or may not depend on your friends' opinion of cool looking maths. *I'd be impressed at least. Anyway, here it is:*

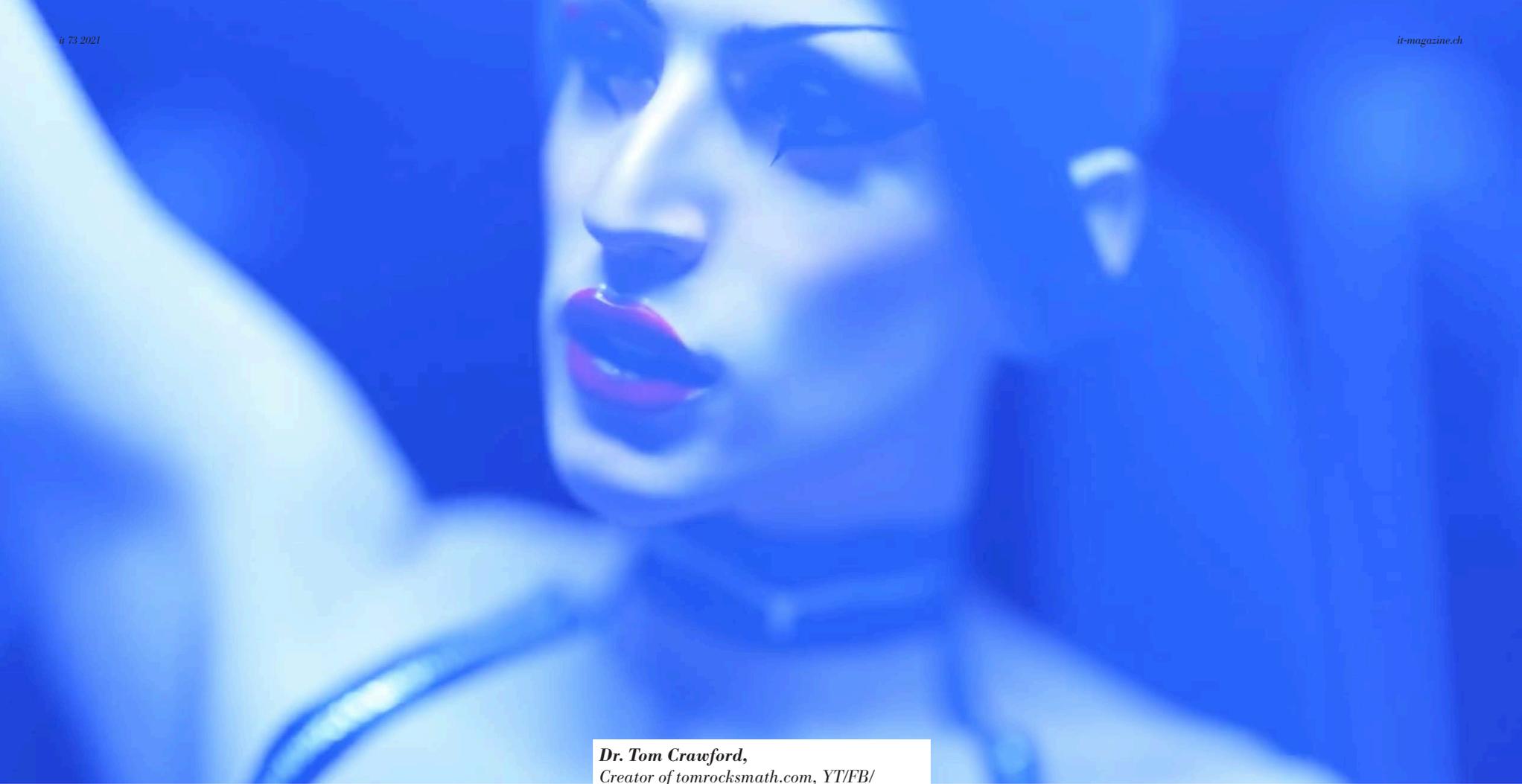
$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} \frac{x^{s-1}}{e^x - 1} dx$$



Beyond how cool it looks written down, we don't really need to know any more about it. *The Riemann Hypothesis* basically says here are some properties about this *Riemann function*: are they true or not? If they are, and most mathematicians think they will be, then it will mean that we understand the location of the prime numbers much better and most importantly we know where to look to find them.

So why do we even care where prime numbers are on the number line? This is the million-dollar question, quite literally. When you use your credit card online, the details are encrypted with a code made up from a number with 200 or so digits to protect you from hackers. Even if they intercept the signal, it just looks like nonsense. The only way to decrypt the code is to know which two prime numbers multiply together to make the 200-digit number. Breaking a number down into prime number factors like we did above for 42 isn't too tricky, because it's a small number. Once the numbers get larger, the game changes. Even the most powerful computers can only work out the prime factors of numbers up to 100 digits in length – so with 200 digits your credit card details are *'as safe as houses'*.

Current methods of online security work precisely because we don't understand the prime numbers. If you were to solve the Riemann Hypothesis you wouldn't just get the million-dollar prize, you'd also be able to steal the credit card details of everyone on the internet... I am absolutely not saying you should do this, just you know, you probably could if you were that way inclined.



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