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Game Theory

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1. Introduction

We live in an interconnected world. We, as humans, make a lot of decisions involving other people consciously and subconsciously. A key aspects of the decisions we make is to maximise our interest or others' interest. Game theory is a branch of mathematics that does just that, it helps us to optimise our strategy by logically analyse every possible outcomes. More formally, it 'provides tools for analyzing situations in which parties, called players, make decisions that are interdependent.(1)

It was extensively developed in the 1940s to 1950s. John Von Neumann was often considered to be the founding father of Game Theory. Nowadays, Game Theory has became an irreplaceable part in our daily lives: it is used by Firms to decide the optimum strategy, it is used by governments to evaluate international relations, it is used for stock trading...etc.

1.1. *The Underlying Assumptions of Game Theory*

One of the important feature of Game Theory is that it only studies non-parametric situations. For example, playing chess in a non-parametric situation: every decision you made taking into account what your opponents will do. Your opponent will also anticipate your move. Throwing a javelin is a parametric situation. You only have to consider how you throw it. The surroundings, for example, gravity will not change just because you decide to throw it with your left hand.

Another fundamental feature of gaming theory is that it assumes all agents are rational and will maximise their utility in all circumstances. (2) Utility is often an index in Game Theory. For example, if I prefer to eat Chinese food over Mexican food over Indian food. I might assign an index of preference to each item. E.g. 10 for Chinese food, 5 for Mexican food and 2 for Indian food and so on. This does not necessary mean I get twice the utility if I choose to eat Chinese food over Mexican food. It is simply a number. Although many real world agents such as a firm or a government can be modelled as rational and only maximise its own welfare, it is not always the case. People will often act in a way to ensure fairness and happiness of others at the cost of some of their own utility.

2. Examples and strategies in Game Theory

2.1. *Dominance*

In Game Theory, we often will draw a table like this: The leftmost column is all the options player 1 has (which is A,B,C), the top row is all the options player 2 has (D,E,F). The table contains all the possible outcomes. Each row contains a pair of numbers: (a,b), a refers to the utility of player 1 whereas b refers to the utility of player 2.

Dominance can be classified into strictly dominant or weakly dominant. When the payoff of all the outcomes(utility) is higher for option a is higher than b, a strictly dominates b. In the example above, strategy C strictly dominates A and B because: (4>2,3) , (7>2,1), (9>3,4). (3) When the payoff of

	D	E	F
A	3,?	2,?	3,?
B	2,?	1,?	3,?
C	4,?	7,?	9,?

all the outcomes(utility) is higher or **equal** for option a than b, a weakly dominates b. In the example above, A weakly dominates B because 3>2, 2>1 and 3=3. For a rational player, eliminate A and B will be the best choice since C strictly dominates them. In Game Theory, we will only consider the best payoff of the strategies, so in order to eliminates ineffective strategies, we can use IDSDS.

2.2. *Iterated Deletion of Strictly Dominated Strategies*

Iterated Deletion of Striclty Dominated Stategies (IDSDS) refers to a series of steps to remove strategies that are dominated by others. After this procedures, the strategy profile can be simplified or even give a single best outcome.

	H	I	J	K	L
A	1,4	3,10	2,8	4,7	3,4
B	0,4	2,5	1,2	0,3	0,2
C	2,8	4,4	3,6	4,5	1,4

TABLE 1 *2.2.1*

As you can see, player 1 would first delete strategy B because A strictly dominates B. Player 2 would also delete strategy L because K strictly dominates L. This leaves a simplified table

	H	I	J	K
A	1,4	3,10	2,8	4,7
C	2,8	4,4	3,6	4,5

TABLE 2 *2.2.2*

In table 2.2.2, strategy J strictly dominates K and H, so we can delete K and H. For player 1, strategy C strictly dominates A so A is eliminated. Now we are left with (4,4) and (3,6). In order to maximise utility, player 2 will always deploy J, so the final outcome of the game is (3,6) in which player 1 uses C and player 2 uses J.

2.3. *A classic example: The Prisoner's dilemma*

The dilemma is as follows: you and your partner committed a crime. The police offers 1 month in jail if both of you keep quiet during interrogation; if one confesses and the other one keeps quiet, the one who confesses gets no jail time while the other will spend 12 month in jail; if you and your partner both confess, you will have 9 months of jail time. To summarise the utility in to a table: (-1 means 1 month in jail)

	Keep Quiet	Confess
Keep Quiet	-1,-1	-12,0
Confess	0,-12	-9,-9

TABLE 3 *2.3.1: The Dilemma*

This is one of the few examples in Game Theory that can be solved by using the concept of dominance. For player 1, confess will always yield a higher utility: 0¿-1, -9¿-12, this is also true for player 2. This leaves both player the only option which is to confess. The final outcomes would be spending 9 months in jail. Although both of them keeping quiet has a better outcome for them collectively, either of them can adapt a better strategy just by confess. However, this is only the case when you and your partner only care about yourselves[1]. If there is trust between you and your partner, (-1,-1) can be achieved.

2.4. *Nash Equilibrium*

Nash equilibrium, just as the name suggests, is developed by John Nash. He is considered one of the greatest mathematicians in the 20th century, he made contributions to Game Theory ans consequently was awarded Nobel Prize in Economics for this.(4) In a Nash Equilibrium, you will not regret your choice no matter what your opponent does. (3)

Consider the following situations: two cars are driving along a road and they are about to collide head on. They can both decide to stop, which wastes time and power. Either one of them can stop, allowing the other car to pass through. Or otherwise collide head on which is disastrous for both of them. To summarise the outcomes, we can draw the table:

	Go	Stop
Go	-10,-10	2,0
Stop	0,2	-2,-2

TABLE 4 *2.4.1*

Is (-10,-10) a Nash equilibrium? Player 1 would choose to stop and achieve a better outcomes and so can player 2 (from -10 to 0). When player 1 sees player 2 is not going to stop, he/she will definitely choose to stop. Consider (-2,-2): it is also not a Nash equilibrium because when player 1 sees player 2 stoping, he would immediately change his/her strategy to not stop and do better. In both cases, either of the player regrets their decision.

This leaves the final two outcomes which are, in fact, Nash Equilibrium. If player 1 decides to go and player 2 decides to stop, non of the players can benefit from switching their strategy. (2>-2 and 0>-10). This is also true if player 1 stops and player 2 goes.

[1] Game Theory is only concerned with rational individual who maximise their own utility

2.5. *Backwards Induction*

This is an example adapted from 'Game Theory 101'. (5).

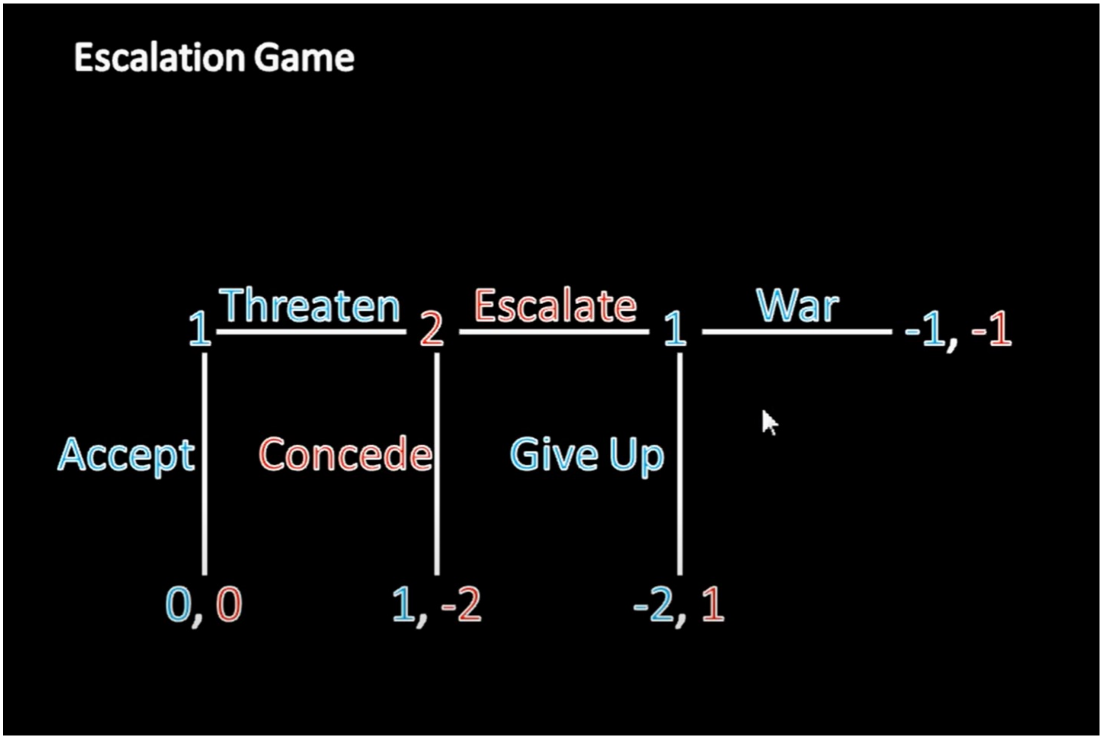

FIG. 1. The 'Escalation Game' adapted from 'Game Theory 101'(5)

Backwards induction is technique used for solving Perfect-information games. This refers to a game in which the player knows all the possibles outcomes before making a move.(3). In the example above, there are 2 countries - one in blue and other one in red, their utilities are also labelled with the same colour. Country 1 can start off by deciding to accept of threaten, and then it is country 2's turn to escalate or concede,and finally, country 1 can decide to go to war or give up. This is a perfect-information game since country 1 have knowledge of all the outcomes. Just as the name suggests, we can start backwards.

In the third junction, country 1 can either to war or give up. Since -1>-2, country 1 will always go to war in that situation. Country 2 knows that country 1 will always go to war, so in the second junction, his utility is either concede (-2) or escalate (-1). This means country 2 will always choose to escalate. Back in the first junction, player 1 understands that if he threathens, country 2 will escalate and hence the outcome is war. So player will choose wisely and choose to **accept** because 0>-1.

3. Conclusion

Game theory is an extremely powerful tool for making decisions in a non-parametric situation. The Escalation game is an oversimplified version of politics that game theory play a part in. However it is also worth noting the limitations of game theory.

3.1. *Limitations of Game Theory*

For example, not everything can be evaluated using an utility scale. Some people believe that a war might benefit the country in the long term, hence giving it a scale of 5. Some people are pacifist and rate war as -5. Especially in politics, it is hard to find a solution to satisfy everyone because we all have different opinions.

In addition, it is not possible to consider all the outcomes throughly. Often, people will only consider the economical impact of a decision, and thereby neglecting the impact it has on society as a whole and the ecosystem. This can potentially leads to endless exploitation of the earth's resource and worsen the effect of Global Warming. In some cases, an effective government is needed to ensure this will not happen.

It is worth remembering that, in real world, possibilities are not limited, you only need to think outside the box. For example, when two businesses are in rivalry, there are more choices than simply reconcile or escalate. It might be beneficial to shift the focus to another target market or spend the money developing a better product. Game theory will yield the best outcome given several strategies. Thinking creatively and generate more strategy is just as important.

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