Much ado about infinity

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Abstract In what follows, I will be discussing what 'infinite' truly means, and illustrate two of the many exciting paradoxes we come across when we flesh out the infinite in detail. I present these paradoxes as arguments against the *actual* infinite, and later refute these arguments, concluding that there doesn't seem to be an issue for those that hold that an actually infinite number of things can exist.

Keywords: actual infinite, potential infinite, paradox, philosophy of mathematics, metaphysics

1 Introduction

It was David Hilbert, arguably one of the greatest mathematicians of all time, that said "No other question has ever moved so profoundly the spirit of man; no other idea has so fruitfully stimulated his intellect; yet no other concept stands in greater need of clarification than that of the infinite." (Newman, 1956). Well, let us here clarify, to the best of our abilities, what we mean by the infinite. We will begin by distinguishing two different concepts; the *potential* infinite and the *actual* infinite. A potential infinite refers to a "collection that is increasing toward infinity but never actually gets there". (Craig, 2008). The actual infinite, then, is "a collection of definite and discrete members whose number is greater than any natural number". (Craig, 2008). The potential infinite is really uncontroversial, the controversiality lies with the actual infinite. Let us move on to exactly why this is the case.

2 Against the 'actual' infinite

The first argument I will outline, against the possibility of an actual infinite, is levied by William Lane Craig. Craig says that absurdities would arise if an actual infinite were to be instantiated in the real world. (Morriston, 2002). He does this by drawing on from a thought experiment proposed by Hilbert called Hilbert's Hotel. Imagine a hotel with an infinite number of rooms, all of which are occupied by at least 1 individual. Hilbert then poses scenarios that seem absurd. The first absurdity is that this seemingly full hotel can somehow accept another guest. An individual arrives at the hotel, which is full. The manager of the hotel tells each current guest to move to room number n+1, where n is the current room being occupied. Now, the first room is available for the new guest to occupy! The second absurdity is that this hotel can accept an infinite amount of new guests. An infinite amount of new guests arrive, presumably dropped off from their infinitely large bus(!), and the manager of the hotel says "No problem, just tell all the current guests to move to the room number 'n x 2', where n is the current room being occupied". Now, all the existing guests are in even numbered rooms, which means that the new guests can take up the odd numbered rooms. We are clearly faced with absurd consequences.

The second paradox is called the 'Grim Reaper Paradox' (GRP) and is derived from José Bernadete but has most recently been thoroughly defended by Alexander Pruss & Robert Koons. Bernadete

himself was adamant that an actual infinite could exist, and in the same book where he lays out the paradox he states "This book is an attack on finitism in all its forms […] A metaphysics of the actual infinite is offered as the solution to the contemporary crisis in the foundations of mathematics" (Bernadete, 1964). Pruss, on the other hand, (2008) says that to resolve the paradox we should abandon the assumption that there can be an actual infinity of objects in existence. Bernadete explains the paradox as such: "A man is shot through the heart during the last half minute by A. B shoots him through the preceding 1/4 minute, C during the 1/8 minute before that, &c. ad infinitum. Assuming that each shot kills instantly (if the man were alive), the man must already be dead before each shot. Thus he cannot be said to have died of a bullet wound" (Bernadete, 1964). Similarly, Koons (2017) presents the paradox; but in somewhat of a more systematic manner: Let's assume that there are an actually infinite number of 'Grim Reapers' who have one function and that is to kill an individual called Fred at a certain time. If Fred is dead, the Grim Reaper (GR) does nothing, and if Fred is alive, the GR kills him. There is a GR who is appointed to kill Fred at 12:01. There is another GR appointed to kill Fred at 12:00:30 (30 seconds after 12) and another at 12:00:15 and another at 12:00:07:30 and so on, *ad infinitum*. Obviously, poor Fred won't survive this situation. For him to survive, he will have to be alive after 12:01, but he can't survive until 12:01 because there is a GR waiting for him at 12:01. And for him to even survive until 12:01, he must be alive at 12:00:30, at which point the other GR would have killed him. No GR has the opportunity to kill him because for every GR, there is another GR waiting to kill Fred. It's impossible that Fred survives and also impossible that any GR kills him!

We are now faced with two seperate paradoxes which have force against the actual infinite.

3 No problem for the infinite

Going back to Craig's view, he presumably wants to formulate the argument like this: (1) if an actual infinite number of things could exist, the Hilbert's Hotel Scenario (HHS) would be possible [p->q], (2) but HHS is not possible [~q], therefore (3) an actual infinite number of things could not exist. [∴~p]. The argument is logically valid, but is it sound? Well, I would reject (2) on the grounds that it seems unmotivated. Why is it the case that the HHS is not possible? Because it is counterintuitive or absurd? Okay, let's grant Craig what he wants and say this is absurd, but surely this can not be enough to justify its impossibility as there are many things that at one point seemed counterintuitive, but now are plausible and true. Take the idea of the globe Earth for example. Let's now move on to the second paradox.

The GRP has more strength, because we come across a logical contradiction (p∧~p), something that doesn't seem to be apparent in the HHS. Granting that there's a logical contradiction, we need to find a way to explain the paradox with the least amount of commitments. The way we can get around the GRP is through the 'Unsatisfiable Pair Diagnosis' (UPD), which is something Nicholas Shackel (2005) coined and applies to Bernadete paradoxes. The basic idea is that the structure of the GRP is such that it creates a contradiction. You can't have both of the two jointly inconsistent propositions: a) there is no first time t, and b) For all t (p at t iff for all t' (if t'<t, then ~p at t')). (Malpass, 2020). But just because they both create a contradiction, doesn't mean we can really rule anything out, for we don't know what is to 'blame'. Maybe, it is just the case that the two propositions are inconsistent and that's it. It doesn't seem obvious to me why we should rule out the possibility that an actually infinite number of things can exist just because we have an inconsistent pair of propositions.

4 Conclusion

In conclusion, then, we have illustrated paradoxes against the actual infinite, such as the Hilbert's Hotel Paradox and the Grim Reaper Paradox, but find that neither of them are successful in the sense that we should rid our minds of the possibility of an actual infinite number of things existing.

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