*N*AVIER *STOKES* *E*QUATIONS

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Millennium Problem number four is my favourite, hands down. I'm probably not supposed to be biased but when you have an equation tattooed on your body the rules change. *The Navier-Stokes equations describe the flow of every fluid you can possibly think of:* rivers, water from a tap, waves, wind, air flow around an aeroplane, ice in glaciers, ketchup, honey dripping off a spoon, blood in your body... I could go on forever.

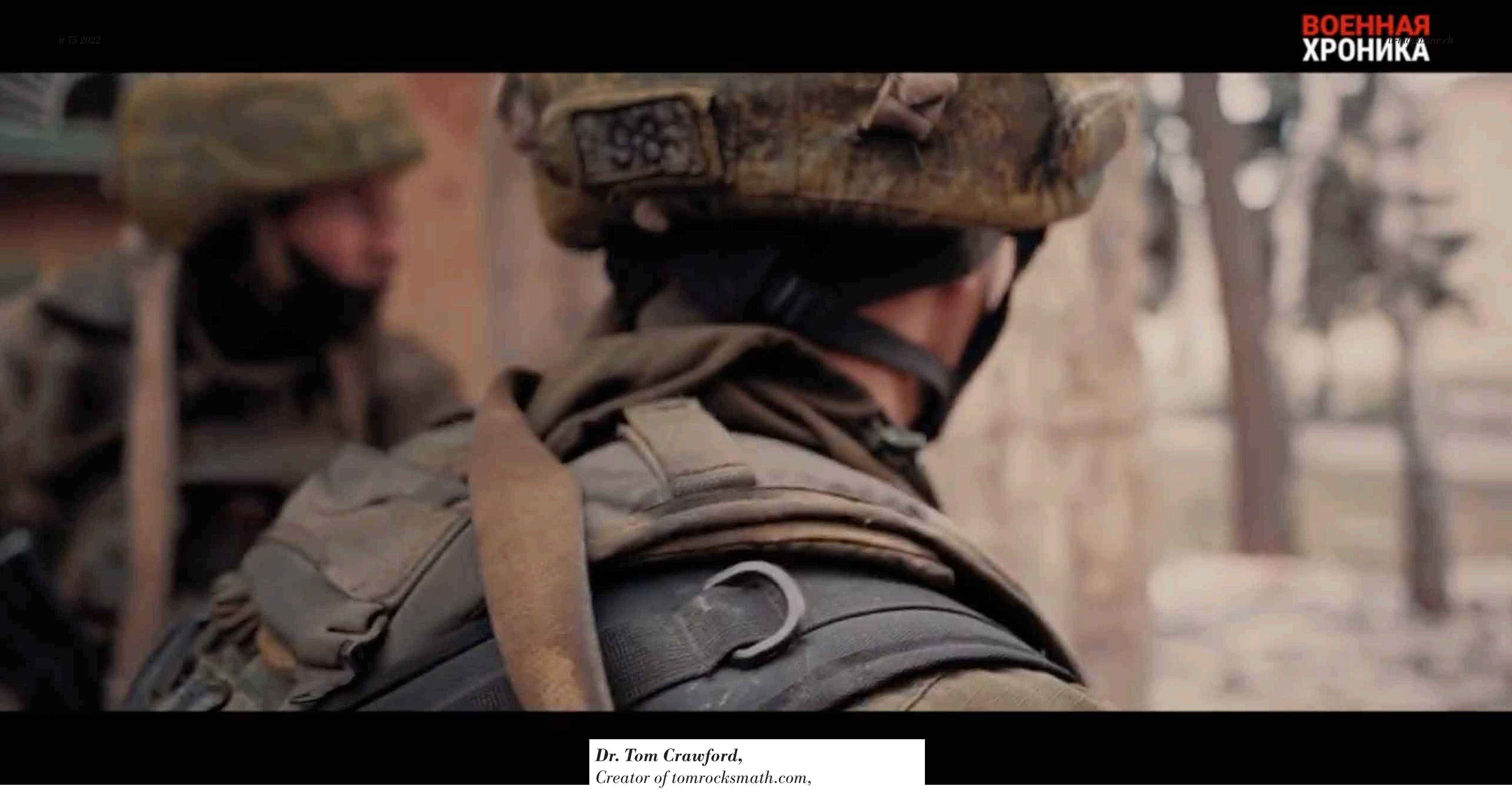
The fact that these equations can do all of this is great – it shows that things in nature behave similarly, and we may actually understand some of it. But, there is a downside. To be able to describe such a variety of different fluids all at once, these equations are super-complex. I'm talking the plot of Inception complex. And just like no-one really understands Inception *(no matter what they might tell you)*, mathematicians don't understand all the little intricacies of the Navier-Stokes equations.

The easiest way to think about it is using what we call a singularity. It might sound complicated but I'm going to explain it step-by-step so stick with me. Start with a number, let's say 2. Divide 1 by 2 and you get 1/2. Now take a smaller number, say 1/4 and divide 1 by it – 'dividing by a fraction is the same as multiplying by it upside down' *(sorry, I'm just hearing the voice of my primary school teacher)*. The answer is 4 though. Now take a smaller number, say 0.1 and divide 1 by it. *You get 10. Take a smaller number 0.01 and divide 1 by it, you get 100. Continue this:* divide 1 by smaller and smaller and smaller numbers and you will get a bigger and bigger and bigger answer. So what happens when you divide 1 by 0? Maths breaks is the answer, but we can think of it as infinity or in our case a singularity.

Singularities occur in nature too with perhaps the most famous example being a black hole. These guys are so complicated that even Stephen Hawking struggles to understand what's going on, which gives you an idea of why singularities are such a nuisance. Going back to the Navier-Stokes equations and the motion of fluids, my favourite example involves bubbles. Let's do a little experiment. Take two circular pieces of wire and holding them close together dip them in soapy water. Imagine those little bottles of bubbles you used to get as a kid, and the plunger thing with the circle bit on the end... that. Well, two of them close together. The idea is that when you hold them close together, dip them in soap and then take them out a bubble will form between the two. It should form a cylinder shape – like the centre of a toilet roll. As you move the two circular wires apart the bubble will stretch and grow taller *(think toilet roll to kitchen roll)*. You can keep moving the wires further and further apart and the bubble gets longer and longer and then POP! You've moved them too far apart and the bubble breaks.

Thinking about this mathematically is a nightmare. It makes sense at first, the wires move further apart and the size of the bubble grows. Increase the distance between the wires and the bubble size increases – a nice simple mathematical relationship. Until you reach the point where the bubble pops. At this instant the increase in the distance between the wires causes a sudden and incredibly fast decrease in the bubble size to zero. It's so fast you can call it infinite. This is your singularity. The video below shows a great example of the experiment I've just described and shows the moment where the bubble size suddenly goes to zero.

As I said above the Navier-Stokes equations model the flow of any and every fluid – this means they describe the bubble popping madness we've just looked at and most importantly the singularity. We don't know how or why or what is going on with these guys – *again, think of black holes* – and that is the Millennium Problem. Can we improve our understanding of these equations? In Lord of the Rings it was one ring to rule them all, in the maths of fluids the Navier-Stokes equations are your ruler... now bow down and make some bubbles.



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