mathematics

Abitur exam 2019

Part A of the exam

Working time: 90 minutes

When processing the tasks**no tools**be used.

For each of the subject areas of analysis, stochastics and geometry, the expert committee selects a task group to work on.**The tasks in part A of the exam belonging to a task group may only be worked on in connection with the tasks in part B of the exam belonging to the same task group.**

_____

_ Name of the examinee

The booklet with the tasks is to be handed in.

Analysis

Task group 1

These questions may only be worked on in connection with the questions belonging to the same group of questions in part B of the examination.

BE

5 **1** The function is given^{2x}<u>It is</u>f:x$_x$

with domainD AND \ 0$_f$ = { }.

Determine the position and nature of the extreme point of the graph of f.

2 Given is the inAND \ 0{ } defined function

$^{-}{}_2{}^{1}$ f:x 1$_x$

, which are the zeros$_1$x 1 = - and

$_2$ x 1 = has. Figure 1 shows the graph of f, which is symmetric about the y-axis. Furthermore, the straight line g with the equationand 3= - given.

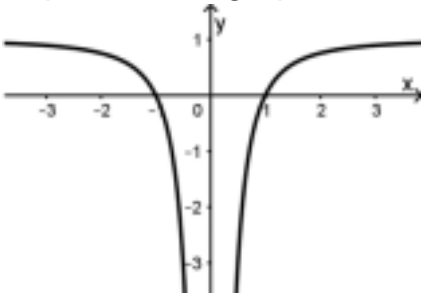


Abb. 1

1 **a)** Show that one of the points where g intersects the graph of f is the x-coordinate$^{1}{}_2$ has.

4 **b)** Determine arithmetically the content of the area enclosed by the graph of f, the x-axis and the line g.

3 Figure 2 below shows the graph a function f.

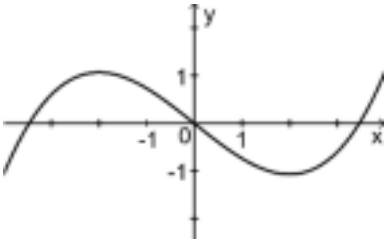


Abb. 2

3 **a)** One of the following graphs I, II and III belongs to the first derivative of f. Write down this graph. Explain that the other two graphs are out of the question.

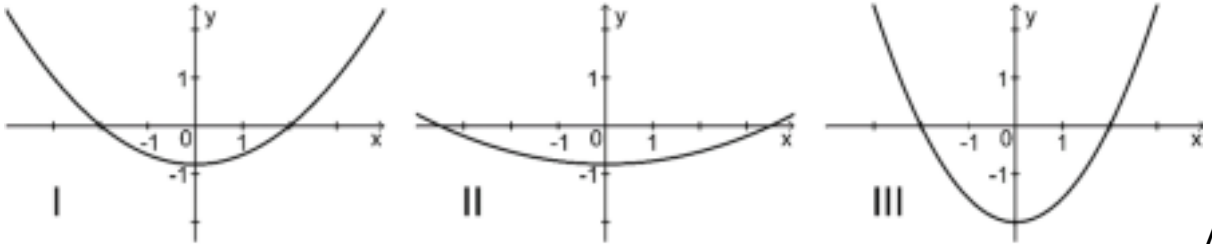


Abb. 3 *2*

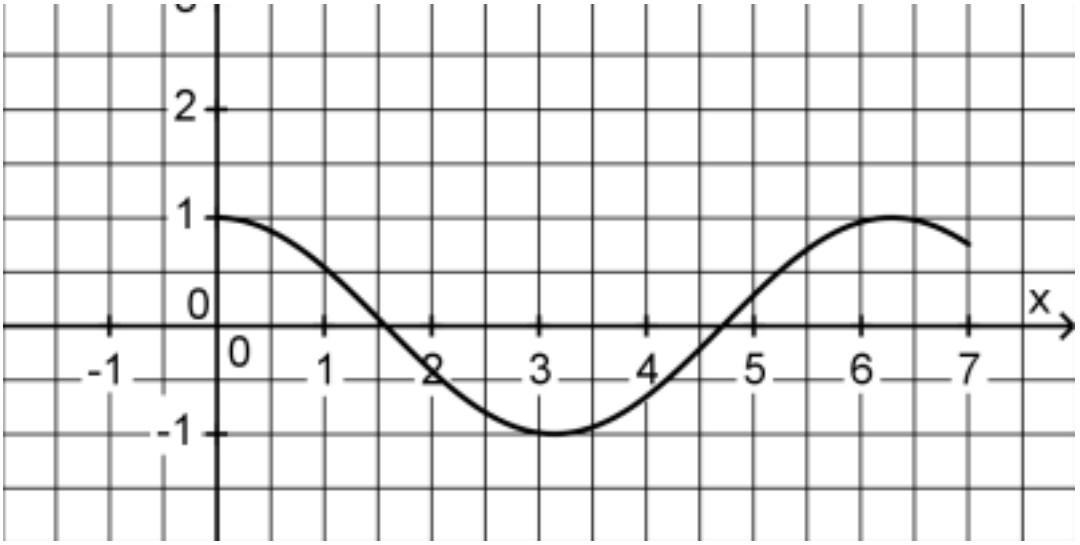
b) The function F is an antiderivative of f. Give the monotonic behavior of F in the interval[ ] 1; 3 at. Justify your statement.*(continued next page)*

3 **4 a)** A set of functions is consideredh$_k$ withk AND^{+} ∈ , which differ only in their

respective domains D_k differentiate.

It appliesh : x cos x $_k$ withD 0;k $_k$ = [ ].

Figure 4 shows the graph of the function h_7 . Give the largest possible value of k such that the associated function h_k is reversible. Draw the graph of the inverse function of for this value of k h_k in Figure 4, paying particular attention to the intersection of the graphs of the function and the inverse function.



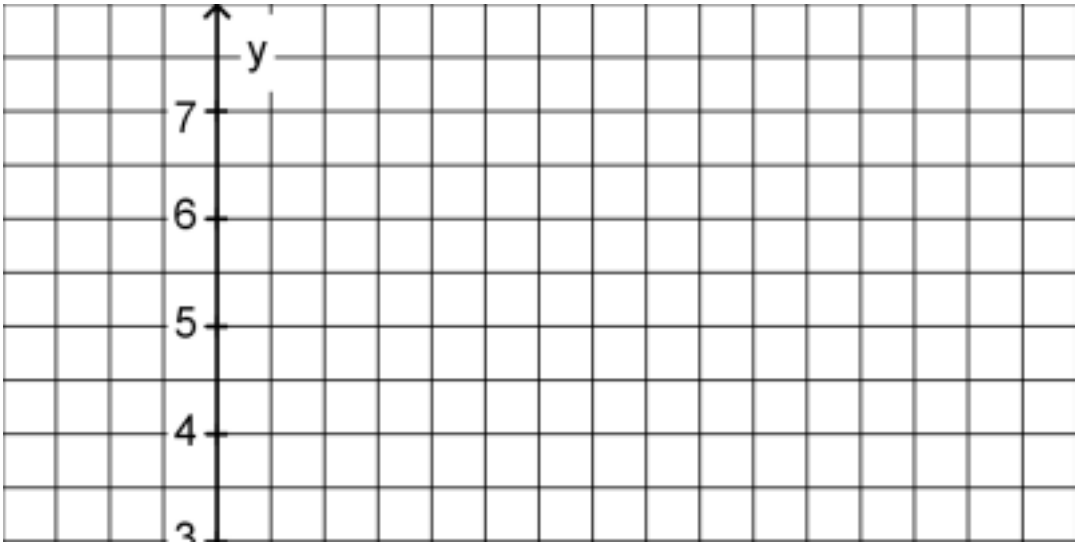


Abb. 4

2 **b)** Enter the term a inANDdefined and invertible function j that satisfies the following condition: The graph of j and the graph of the inverse function of j have no point in common.

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Analysis

Task group 2

These questions may only be worked on in connection with the questions belonging to the same group of questions in part B of the examination.

BE

1 The function is giveng: x 2 - with maximal definition set D.

1 **a)** Enter D.

4 **b)** Find the equation of the tangent to the graph of g at the point(8|g 8( )).

2 Given is the inAND \ 0{ } defined function

- 2 1 f:x 1 x

, which are the zeros 1 x 1 = - and

2 x 1 = has. Figure 1 shows the graph of f, which is symmetric about the y-axis. Furthermore, the straight line g with the equationand 3= - given.

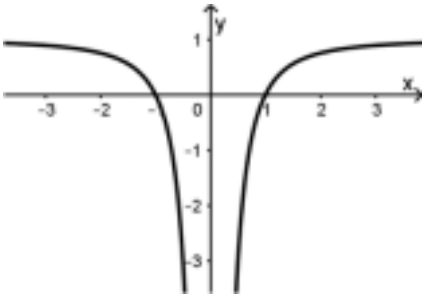

Abb. 1

1 **a)** Show that one of the points where g intersects the graph of f is the x-coordinate$\frac{1}{2}$ has.

4 **b)** Determine arithmetically the content of the area enclosed by the graph of f, the x-axis and the line g.

3 Given is the set of inANDdefined functions- - ^{2} p : x kx 4x 3 $_{k}$ withk AND \ 0∈ { } , whose graphs are parabolas.

2 **a)** Determine the value of k such that the point( ) 2| 3- lies on the associated parabola.

3 **b)** Determine those values of k for which the corresponding functionp_k has no zero.

(continued next page)

4 Figure 2 below shows the graph a function f.

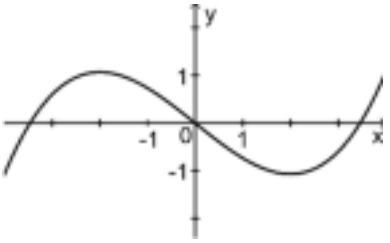

Abb. 2

3 **a)** One of the following graphs I, II and III belongs to the first derivative of f. Write down this graph. Explain that the other two graphs are out of the question.

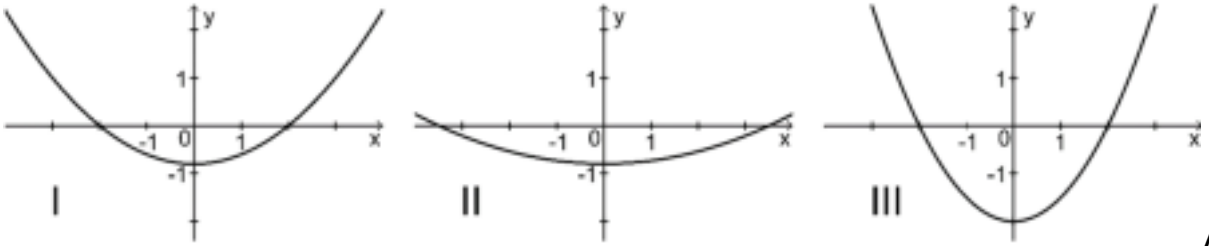

Abb. 3 *2*

b) The function F is an antiderivative of f. Give the monotonic behavior of F in the interval[ ] 1; 3 at. Justify your statement.

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stochastic

Task group 1

These questions may only be worked on in connection with the questions belonging to the same group of questions in part B of the examination.

BE

1 A wheel of fortune consists of five sectors of equal size. One of the sectors is labeled "0", one "1" and one "2"; the other two sectors are labeled "9".

2 **a)** The wheel of fortune is spun four times. Calculate the probability of getting the numbers 2, 0, 1 and 9 in the order given.

3 **b)** The wheel of fortune is spun twice. Find the probability that the sum of the numbers obtained is at least 11.

3 **2** The random variable X can only have the values 1, 4, 9 and 16. Are knownP X 9 0,2 ( ) = = andP X 16 0,1 (= =) as well as the expected valueEX 5 ( ) = . Using an expected value approach, determine the probabilitiesPX 1 (=) andPX 4 (=) .

2 **3** Given is a Bernoulli chain of length n and hit probability p. Explain that to everyonek 0; 1; 2;...; of∈{ } the RelationshipB n; p; k B n; 1 p; n k ( ) = -- ( ) is applicable.

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stochastic

Task group 2

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2 **a)** The wheel of fortune is spun four times. Calculate the probability of getting the numbers 2, 0, 1 and 9 in the order given.

3 **b)** The wheel of fortune is spun twice. Find the probability that the sum of the numbers obtained is at least 11.

2 **2** A binomially distributed random variable X with the parameter value is givenn 5 = . The diagram in Figure 1 shows

the probability valuesPX k ( ) ≤ with
k 0; 1; 2; 3; 4 ∈{ } remove.
Complete the tok 5 = include the probability value in
Diagram. Determine the approximate probability
PX 2 (=).

3 **3** The tree diagram in Figure 2 belongs to a random experiment with stochastically independent events A and B. Determine the probability of event B.

Abb. 1

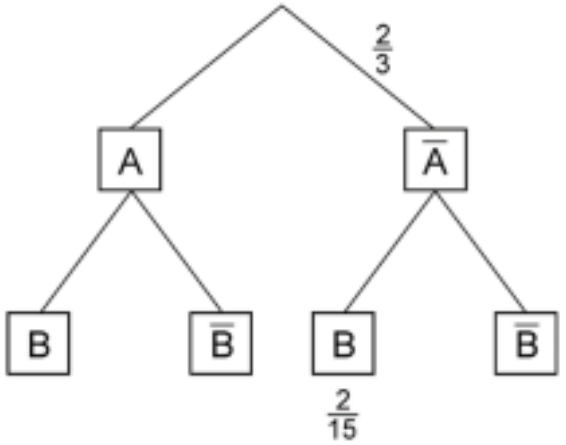

Abb. 2

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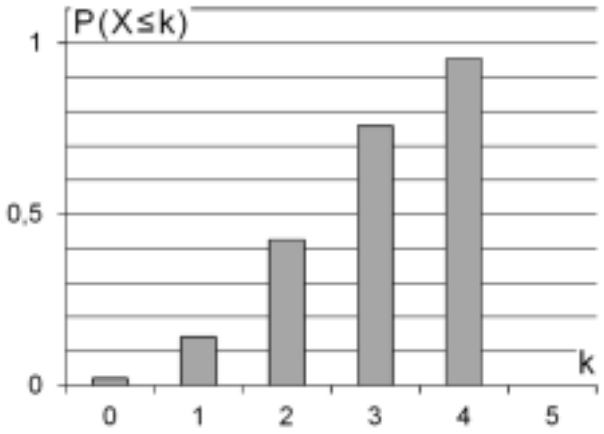

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Geometry

Task group 1

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BE

1 Given is a rectangle ABCD with the verticesA 5| 4| 3 (- -), B 5|4|3 ( ), C 0|4|3 ( ) and d

3 **a)** Find the coordinates of D and give the coordinates of the midpoint M of the segment[ ] ACan.

2 **b)** Explain that the triangles BCM and ABM have the same area without calculating it.

2 **2 a)** The levelE : 3x 2x 2x 6_{123} ++= contains a point whose three coordinates coincide. Determine these coordinates.

3 **b)** Show that the following statement is true: There are infinitely many planes that do not contain a point whose three coordinates are the same.

10

Geometry

Task group 2

These questions may only be worked on in connection with the questions belonging to the same group of questions in part B of the examination.

BE

1 The two balls are given$_1$k with centerM 1| 2| 3 $_1$( ) and Radius 5 as well$_2$ k with centerM 3| 2|1 $_2$ (- -) and radius 5.

2 **a)** Show that yourself$_1$k and$_2$ k cut.

3 **b)** The cut figure of$_1$k and$_2$ k is a circle. Determine the coordinates of the center and the radius of this circle.

2 **2 a)** The levelE : 3x 2x 2x 6_{123} ++= contains a point whose three coordinates coincide. Determine these coordinates.

3 **b)** Show that the following statement is true: There are infinitely many planes that do not contain a point whose three coordinates are the same.

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